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M.A./M.Sc. (Fourth Semester)

EXAMINATION, May-June, 2022

STATISTICS

Paper Second

(Inference-II)

Time : Three Hours]   *[Maximum Marks: 80*

Note- All sections as directed.

Section - A

(Objective/Multiple Type Questions)

(1 mark each)

Note- Attempt all questions. Choose the correct answer.

P.T.O.

1. Let $\phi(x)$ be a test function for testing $H_0: \theta = \theta_0$ versus $H_1: \theta = \theta_1$ for a certain distribution $f(x, \theta)$, Then type-II error of the test is given by

   (A) $E_{\theta_0}[d(x)]$

   (B) $E_{\theta_1}[1-d(x)]$

   (C) $E_{\theta_0}[1-d(x)]$

   (D) $E_{\theta_1}[d(x)]$

2. Let X has cauchy distribution with pdf $f(x,\theta) = \dfrac{1}{\pi[1+(x-\theta)^2]}, \ -\infty < x, \theta < \infty$. To test $H_0: \theta = 0$ versus $H_1: \theta = 1$ the critical region is given as $1 \le x \le 3$, then size of the test is

   (A) $\dfrac{\tan^{-1}3}{\pi} - \dfrac{1}{4}$

   (B) $\tan^{-1}3 - \dfrac{\pi}{4}$

   (C) $\dfrac{1}{4} - \dfrac{\tan^{-1}3}{\pi}$

   (D) $\dfrac{\pi}{4} - \tan^{-1}3$

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3. For any test to test H: $\theta \leq 0.5$ versus K: $\theta > 0.5$, if size of the best test is 0.05 and power is 0.85 and test is called

   (A) UMP biased

   (B) UMP unbiased

   (C) MP biased

   (D) MP unbiased

4. Consider the following statements for testing simple null versus simple alternative hypothesis.

   I: The NP lemma does require the sample values to be independent and identically distributed

   II: The distribution specified under null and alternative hypothesis must belong to the same family.

   The correct answer is-

   (A) Both I and II are true

   (B) Only I is true

   (C) Only II is true

   (D) Both I and II are false

5. Let $x_1, x_2,...,x_n$ be a random sample from $N(O,O^2)$ normal distribution. The distribution possess MLP property in statistics $T(x)=$

   (A) $\sum_{1}^{n} x_i$

   (B) $\sum_{1}^{n} x_i^2$

   (C) $\sum_{1}^{n} (x_i - \bar{x})^2$

   (D) None of these

6. To test H: $\theta \leq \theta_0$ versus K: $\theta > \theta_0$ based on a random sample of size n from $f(x,\theta)=e^{-(x-\theta)}$, $x \geq \theta$, the test statistic used to obtain UMP test is

   (A) $\sum_{1}^{n} x_i$

   (B) $\sum_{1}^{n} x_i^2$

   (C) $X_{(1)}$

   (D) $X_{(n)}$

7. Necessary condition(s) for test $\phi(x)$ to be α-similar to test H: $\theta \in \Omega_H$ versus K: $\theta \in \Omega_K$ is(are)

I. $\phi(x)$ must be unbiased

II. power function of the test is continuous in θ.

The correct answer is-

(A) Only I

(B) Only II

(C) Neither I nor II

(D) Both I and II

8. Necessary condition for existence of UMP test in general to test H: $\theta = \theta_0$ versus K: $\theta \neq \theta_0$ under regularity condition is

(A) $\left. \dfrac{\partial \log L(\theta \mid x)}{\partial \theta} \right\}_{\theta = \theta_0} = 0$

(B) $\left. \dfrac{\partial \log L(\theta \mid x)}{\partial \theta} \right\}_{\theta = \theta_1} = 0$

(C) $\left. \dfrac{\partial \log L(\theta \mid x)}{\partial \theta} \right\}_{\theta = \theta_0} = $ constaint independent of $\underline{x}$

(D) $\left. \dfrac{\partial^2 \log L(\theta \mid \underline{x})}{\partial \theta} \right\}_{\theta = \theta_0} = 0$

P.T.O.

9. The range of size of any test is

(A) 1 to n

(B) 0 to ∞

(C) 0 to 1

(D) -1 to 1

10. For LRT consider the following statements-

I: The large value of criterion $\lambda(\underline{x})$ will test to reject H_0

II: While formating LRT, there is no severe assumption on the nature of the probability density function

The correct answer is-

(A) I but not II

(B) II but not I

(C) Both I and II are true

(D) Both I and II one false

11. 'SPRT' stands for

   (A) Simple probability ratio test

   (B) Single probability ratio test

   (C) Sequential probability ratio test

   (D) Sequential probability random test

12. Criterion for no decision at m^{th} stage in SPRT with B<A

   (A) $\lambda^m(x) \le B$

   (B) $\lambda_m(x) \ge A$

   (C) $B < \lambda_m(x) < A$

   (D) $A < \lambda_m(x) < B$

13. In SPRT with B<A, under usual notations, the OC function of the test is

   (A) $\dfrac{1 - B^{h(\theta)}}{A^{h(\theta)} - B^{h(\theta)}}$

   (B) $\dfrac{A^{h(\theta)} - B^{h(\theta)}}{1 - B^{h(\theta)}}$

   (C) $\dfrac{1 - A^{h(\theta)}}{B^{h(\theta)} - A^{h(\theta)}}$

   (D) $\dfrac{B^{h(\theta)} - A^{h(\theta)}}{1 - B^{h(\theta)}}$

14. In usual notations, ASN for SPRT with B<A is-

   (A) $\dfrac{A.L(\theta) + B.(1 - L(\theta))}{E_\theta(z)}$

   (B) $\dfrac{L(\theta).\log A + (1 - L(\theta)).\log B}{E_\theta(z)}$

   (C) $\dfrac{L(\theta).\log B + (1 - L(\theta)).\log A}{E_\theta(z)}$

   (D) $\dfrac{L(\theta).\log A - (1 - L(\theta)).\log B}{E_\theta(z)}$

15. When null hypothesis is true and $h(\theta) = 1$, the OC function of SPRT becomes

   (A) α

   (B) β

   (C) $1 - \alpha$

   (D) $1 - \beta$

16. An alternative non parametric test for testing means of two population compared to parametric t-test is

   (A) Mann-Whitney $\cup$ test

   (B) Run test

   (C) Sign test

   (D) None of these

17. In Mann-Whitney $\cup$ test for two samples of sizes n_1 and n_2, $E(\cup)=$

(A) $\dfrac{n_1 n_2}{2} + 1$

(B) $\dfrac{n_1 + n_2}{2}$

(C) $\dfrac{n_1}{n_1 + n_2}$

(D) $\dfrac{n_1 n_2}{2}$

18. Test used for goodness of fit with very small sample size is

(A) Sign test

(B) Run test

(C) K-S test

(D) Median test

19. The differences of 5 pair values are given below

i:	1	2	3	4	5
Di:	-5	-1	1	1	5

Then the value of $T^+ =$

(A) 7   (B) 26.5

(C) 8.5   (D) 9

P.T.O.

20. Total number of runs of symbol 'F' in the following sequence is

MMFFFMMFFMF

(A) 11

(B) 5

(C) 6

(D) 3

Section - B

(Very Short Answer Type Question)

(2 marks each)

Note: -Attempt all questions. Answer precisely using 2-3 sentences only.

1. Define randomised test function.

2. Define power of the test.

3. Define unbiased test.

4. State MLR property.

5. State LRT criterion.

6. State the distribution used in sign test.

7. State stopping bounds with approrimate values in SPRT

8. When median test is used?

Section-C

(Short Answer Type questions)

(3 marks each)

Note- Attempt all questions. Write your answer using 75 words.

1. Let X has binomial $b(2,\theta)$ distribution. To test $H: \theta = \frac{1}{2}$ versus $K: \theta = \frac{3}{4}$ a randomised test $\phi(x)$ bio is defined as $d(x) = e^{-x}$, $x \in \mathcal{X}$. Then obtain probability of type-I and type-II errors.

2. Let $X \sim U(\theta, \theta H)$ uniform distribution. To test $H: \theta = 0$ versus $K: \theta = 1$, the test function is $d(x) = 1$ if $x > 0.95$ and O otherwise. Obtain size and power of the test.

P.T.O.

3. Prove that every UMP test if exists, it is UMPU.

4. State important properties of LRT.

5. Describe sign test.

6. Describe Run test.

7. Obtain ASN function of SPRT

8. In SPRT, with usual notations show that $B \geq \frac{\beta}{1-\alpha}$

Section-D

(Long Answer Type Questions)

(5 marks each)

Note- Attempt all questions. Write your answer using 150 words.

1. Let x_1 and x_2 be iid poisson (λ) random variables. Consider non randomised test function

$$\phi_1(x_1) = \begin{cases} 1, & if\ x_1 = 2 \\ 0, & otherwise \end{cases}$$

to test $H: \lambda \leq 1$ versus $K: \lambda > 1$

Hence, deduce randomised test function

$E[\phi_1(x_1)|x_1+x_2=t]$. Also obtain its size.

OR

Let $x_1, x_2, ..., x_n$ be a random sample from $\cup(0, \theta), \theta > 0$ uniform distribution. Derive UMP test for testing

H: $\theta \leq \theta_0$ versus K; $\theta > \theta_0$. Also, obtain power function of the test.

2. Let $x_1 \sim N(\mu_1, 1)$ and $x_2 \sim N(\mu_2, 1)$. To test

H; $\mu_1 = \mu_2$ versus K: $\mu_1 \neq \mu_2$ derive level α LRT.

OR

Let $X_i \sim IN(\mu, \sigma^2)$, $i = 1, 2, ..., n$. Consider a problem for testing H: $\mu = \mu_0$ versus K: $\mu \neq \mu_0, \mu$ and σ^2 both are unknown. Obtain LRT of level α.

3. Obtain general form of O.C. function of SPRT.

OR

Let $x_1, x_2, ...$ be a sequence of observations from uniform U(0, θ), $\theta > 0$ distribution. Describe SPRT procedure to test H: $\theta = \theta_0$ versus K: $\theta = \theta_1 (\theta_1 > \theta_0)$.

4. Obtain probability distribution of T^+ in case of a sample of size 4 with all distinct values.

OR

Describe chi-square test of goodness of fit.